New developments

Definitions

In a system where a planet or probe (P2) is orbiting another planet (P1), orbiting a sun.

y - P1 apparent position (the latitude, positive being north and negative-south) as observed from P2

o - P2 orbital inclination

x - P2 axial tilt

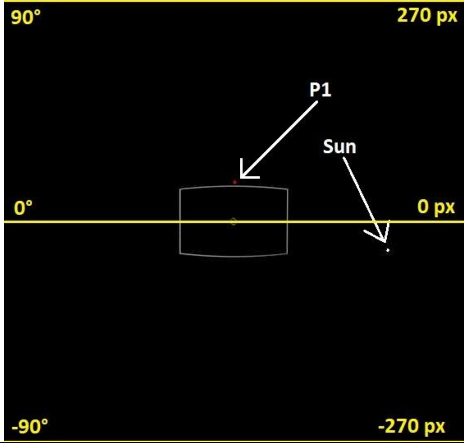
b - P2 axial tilt direction

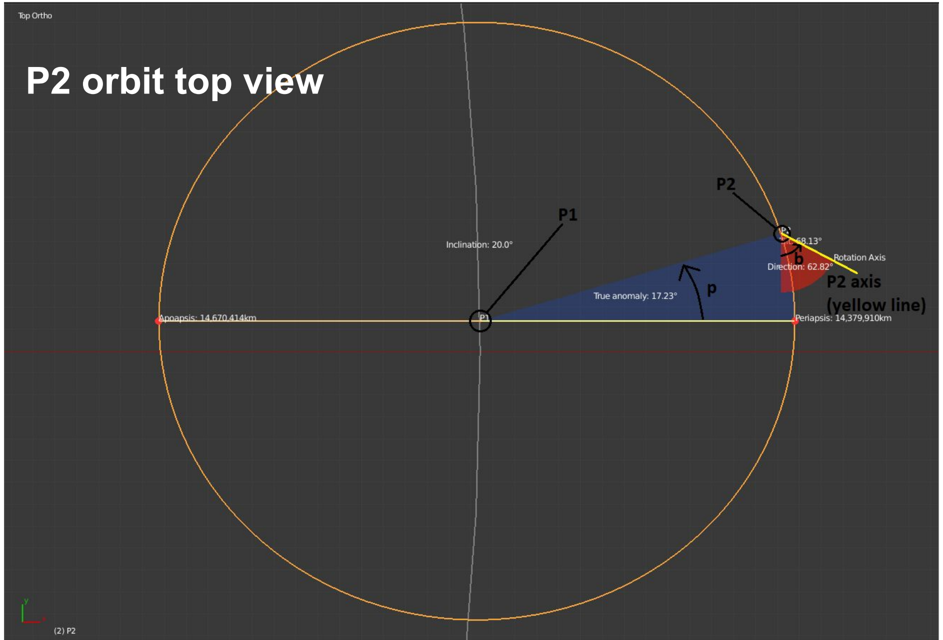
p - P2’s position on orbit (true anomaly)

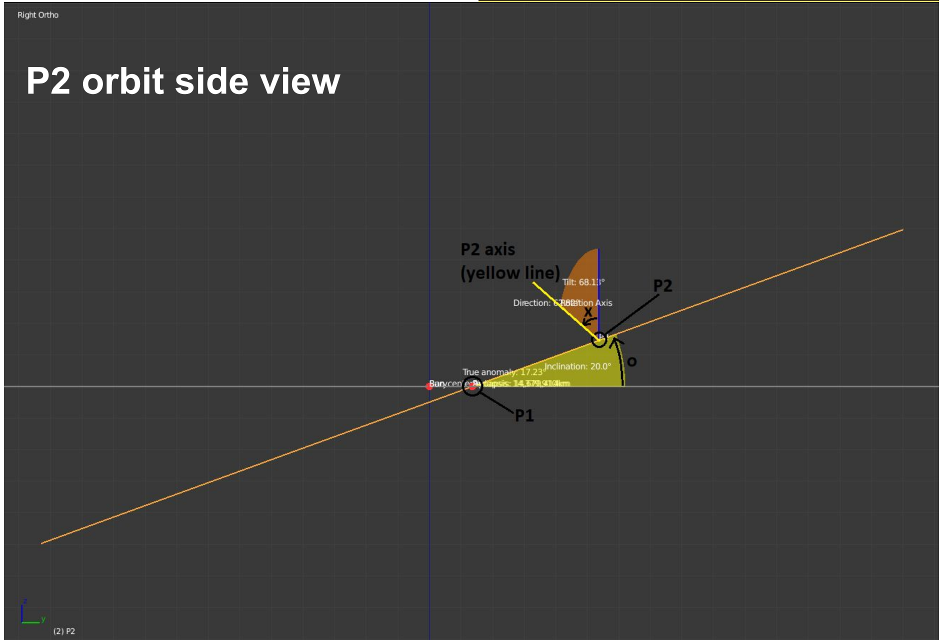
Everything is in radians

Definitions

360 degree view from P2







Apparent motion of P1

Lets mark the middle of P2’s view as 0 degrees, the top as 90 degrees, and the bottom as -90 degrees. Graphing the apparent position of P1 as a function of P2 position on orbit (true anomaly), using these coordinates draws a sine wave. The period of this sine wave is constant: 180 degrees. The amplitude and phase shift of the function change as P2’s axial tilt, axial tilt direction, and orbital inclination change.

y = amplitude * sin(p - phase shift)

Derivation of amplitude

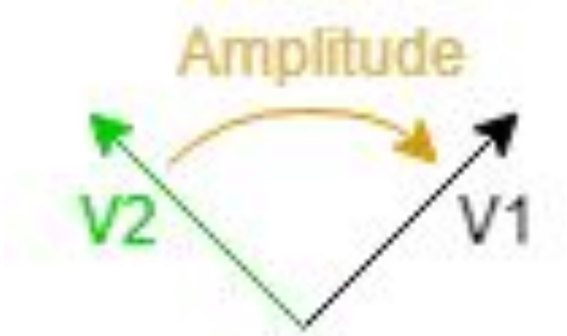
The amplitude is the angle between the line from P1 to P2 at the point where P1 appears to be the farthest from the center, and P2's axis. This angle can be found by drawing a unit vector along these two lines.

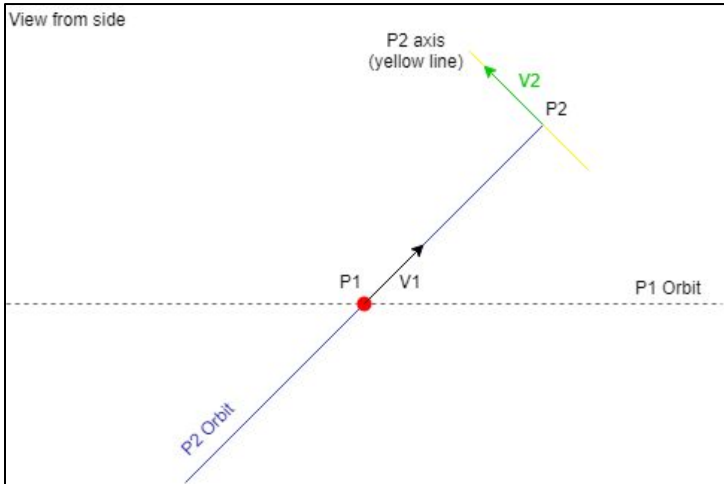
The vector along P2's axis (V2): $(\sin x \times \cos b, \quad \sin x \times \sin b, \quad \cos x)$

The vector along the line from P1 to P2 (V1): $(\sin o\,, \quad 0, \quad \cos o)$

The amplitude (angle between both of these vectors):

$$\cos^{-1}(\cos o \times \cos x + \sin o \times \sin x \times \cos b)$$





Derivation of phase shift

The phase shift is determined by the position on orbit (true anomaly) of P2 where P1 appears to be in the middle of P2's view. This point is the point where the effects of P2 orbital inclination and effects of P2 axial tilt and tilt direction cancel out. We can write an equation, marking the phase shift as S.

$$x \times \sin(S - b) - o \times \sin(S) = 0$$

Green - effect of P2 axial tilt and tilt direction

Yellow - effect of P2 orbital inclination

Solving this equation, we get:

$$S = \cot^{-1}\left(\cot(b) - \frac{o}{x \times \sin(b)}\right)$$

Equation giving P1 apparent position

$$y = \cos^{-1}(\cos o \times \cos x + \sin o \times \sin x \times \cos b) \times \sin\left(p - \cot^{-1}\left(\cot b - \frac{o}{x \times \sin b}\right)\right)$$

Blue - amplitude

Red - phase shift

Testing

Y axis - P1 apparent position (above the center of P2’s view being positive, and below-negative) in pixels
X axis - true anomaly in degrees
Points marked with letters - data from experiments
Line - function drawn by the equation